DEFORMATION OF LIQUID LINE WITH THE COLLISION OF JETS

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With the study of the collision of metallic plates accelerated by an explosion, investigation of plastic deformations in the collision zone is of great importance. In [1] a method is expounded for investigations of deformation with explosion welding under conditions of wave formation; the method consists of the pressing of a thin wire into the metallic plate. With collision of the plates, the flowing metal carries the wire with it; the change in the form of the latter makes it possible to evaluate the character of the plastic deformation in the collision zone. From an investigation of the deformation of the wires, such important characteristics as the viscosity of the metals can be found. The method set forth in [1] is one of the few means of investigating the deformation of metals with high-speed collisions. The difficulties in the investigation are bound up with the small times of the process and with the high pressures, developing with an explosion and demolishing the experimental unit. In [1], from an analysis of viscous flow during the collision of plates, a dependence of the shift on the distance to the interface between the materials, described by a parabola, is derived. It is noted that, near the interface, the theoretical and experimental results differ considerably. It is important to make an attempt at a theoretical analysis of the wire deformation if, with the collision of metallic plates, a jet is formed and the interface between the materials is even. In this case, the flow differs strongly from the flow under conditions of wave formation [2], and near the interface the flow cannot be described by a parabolic dependence; this dependence will probably be exponential. To describe flow with the formation of a jet, in [3] a model of an ideal liquid is employed. Since the analysis of a collision between viscous jets with a free boundary is bound up with serious difficulties, in the first approximation it is advisable to consider the deformation of the liquid line with the deformation of ideal jets.

§1. Let there be a region of space, occupied by a plane steady-state flow of an ideal liquid, and, at the moment of time t_0, let there be isolated a volume of the liquid, bounded by the closed curve of the lines

$$f_0(z_0) = c, \tag{1.1}$$

where z_0 is a complex coordinate; c is a complex constant. With the passage of time, the isolated liquid volume will change its form, being displaced along with the flow as a whole. We pose the problem of finding the form of the liquid volume at some moment of time t. The coordinate of an isolated Lagrangian particle with motion satisfies the integral equation

$$z(t) = z_0 + \int_{t_0}^{t} \bar{\zeta}(z(t))\,dt, \tag{1.2}$$

where $\zeta(z)$ is the complex-conjugate velocity, defined in terms of the complex potential $w = \varphi + i\psi$,

$$dw/dz = \zeta. \tag{1.3}$$

We call the liquid line f(z, t) the boundary of the volume. Using (1.2), we write

$$f(z, t) = f_0\left(z - \int_{t_0}^{t} \bar{\zeta}(z)\,dt\right) = c. \tag{1.4}$$

Expression (1.4) for the form of the liquid volume at the moment of time t, written in general form, cannot be used for the solution of actual problems. This is connected with the fact that, with a given form of the function $f_0(z_0)$, the function f(z, t) can be written out when the integral equation (1.2) has been solved. The solution of (1.1) is not possible with any function $\zeta(z)$, and the function $\bar{\zeta}(z)$ itself can be written in explicit form only for the simplest cases of steady-state flow.

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It is more convenient to consider the motion of the liquid volume in the plane w. Since w(z) is an analytical function, occupied by the liquid volume, the region of the physical plane corresponds to some region in the plane w, bounded by the closed curve $f(z(w), t) = F(w, t) = c$, and motion in the physical plane is accompanied by motion in the plane w.

We represent the boundary $F(w, t) = c$ as consisting of several single-valued curves $\Phi_i(\psi)$. For solution of the problem of finding the form of the liquid volume it is obvious that it is sufficient to be able to determine the form of an arbitrary single-valued liquid $\Phi(\psi)$ at an arbitrary moment of time and to know the law of the transformation of z(w). In all that follows, up to the final result, we consider the problem of the deformation of the liquid line, since there is no difficulty in constructing a closed volume from several single-valued lines $\Phi_i(\psi)$.

We determine the velocity of a Lagrangian particle in the plane w:

$$\frac{dw}{dt} = \frac{dw}{dz}\frac{dz}{dt} = |\zeta(\varphi, \psi)|^2 = \frac{d\varphi}{dt}, \tag{1.5}$$

from which it follows that the motion in the plane w takes place along straight horizontal lines and, at first, with motion, the single-valued function $\Phi(\psi)$ remains single-valued.

We introduce the function $\xi = \omega - i\theta$, in accordance with the condition $\xi = \ln \zeta$. Since $w(\xi)$ is an analytical function,

$$\frac{\partial\varphi}{\partial\theta} = \frac{\partial\psi}{\partial\omega}, \quad \frac{\partial\varphi}{\partial\omega} = -\frac{\partial\psi}{\partial\theta}. \tag{1.6}$$

We derive an equation describing the change in the derivative $d\Phi(\psi)/d\psi$ with motion. Differentiating (1.5) with respect to ψ, we can obtain the equation

$$\frac{d}{d\psi}\frac{d\Phi(\psi)}{dt} = 2\left(\frac{\partial\omega}{\partial\varphi}\frac{d\Phi}{d\psi} + \frac{\partial\omega}{\partial\psi}\right)e^{2\omega}.$$

Changing the order of differentiation in the left-hand part of the equation, substituting $e^{2\omega}d/d\varphi$ in place of d/dt and denoting $d\Phi(\psi)/d\psi$ by $\delta(\varphi)$, we arrive at the ordinary differential equation

$$\frac{d\delta(\varphi)}{d\varphi} = 2\left(\frac{\partial\omega}{\partial\varphi}\delta(\varphi) + \frac{\partial\omega}{\partial\psi}\right). \tag{1.7}$$

Solution of (1.7) by the method of the variation of constants gives the expression

$$\delta(\varphi) - \delta(\varphi_0) = 2e^{2\omega}\int_{\varphi_0}^{\varphi}\frac{\partial\omega}{\partial\psi}e^{-2\omega}d\varphi. \tag{1.8}$$

In view of the fact that it is always possible to reconstitute the sought function from a known derivative, (1.8), in principle, gives an answer to the posed problem, finding the form of the liquid line during the process of the motion. The result (1.8) is applicable for any given plane potential flow of an ideal liquid, where the analytical function $w(\xi)$ is given by a mutually single-valued mapping of regions of the flow on the planes w and ξ.

§2. In the case of motion within infinite limits with the collision of jets, using (1.3) and (1.8), we can write

$$\varphi_0 = -\infty, \quad \varphi = \infty, \quad e^{2\omega(\pm\infty, \psi)} \equiv V^2.$$

Then

$$\delta(\infty) = \delta(-\infty) + 2V^2\int_{-\infty}^{\infty}\frac{\partial\omega}{\partial\psi}e^{-2\omega}d\varphi. \tag{2.1}$$

From (2.1) an expression is obtained for the final form of the liquid line

$$\Phi(\psi)_{\infty} = \Phi(\psi)_{-\infty} + 2V^2\int_{\psi_0}^{\psi}\int_{-\infty}^{\infty}\frac{\partial\omega}{\partial\psi}e^{-2\omega}d\varphi d\psi \tag{2.2}$$

with the condition $\Phi(\psi_0)_{-\infty} = 0$ (ψ_0 is the value of the stream function at the free boundary).

For the practical use of (2.2), it is convenient to go over in (2.1) to integration in the plane (ω, θ). Since the integration in (2.1) is carried out with $\psi = \text{const}$,

$$\frac{\partial\psi}{\partial\omega}d\omega + \frac{\partial\psi}{\partial\theta}d\theta = 0$$

and

$$\frac{d\omega}{d\theta} = -\frac{\partial\psi/\partial\theta}{\partial\psi/\partial\omega}. \tag{2.3}$$

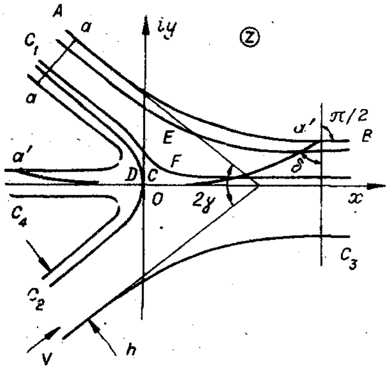

Fig. 1

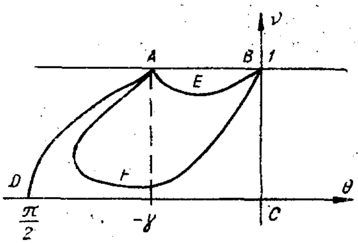

Fig. 2

We obtain an expression for $d\varphi$ in the form

$$d\varphi=\frac{\partial\varphi}{\partial\theta}\left(\frac{d\omega}{d\theta}\frac{\partial\varphi/\partial\omega}{\partial\varphi/\partial\theta}+1\right)d\theta,$$

and, taking account of (1.6), (2.3), we finally obtain

$$d\varphi=\frac{\partial\psi}{\partial\omega}\left(1+\left(\frac{d\omega}{d\theta}\right)^2\right)d\theta. \tag{2.4}$$

Using the Jacobian of the transformation, we find the value of $(\partial\omega/\partial\psi)(\partial\psi/\partial\omega)$:

$$\frac{\partial(\omega,\varphi)}{\partial(\psi,\varphi)}\frac{\partial(\psi,\theta)}{\partial(\omega,\theta)}=\frac{\partial(\omega,\theta)}{\partial(\psi,\varphi)}\frac{\partial(\omega,\varphi)}{\partial(\omega,\theta)}\frac{\partial(\psi,\theta)}{\partial(\omega,\theta)}=\frac{1}{\dfrac{\partial(\varphi,\psi)}{\partial(\theta,\omega)}}\left(\frac{\partial\psi}{\partial\omega}\right)^2,$$

$$\frac{\partial(\varphi,\psi)}{\partial(\theta,\omega)}=\begin{vmatrix}\dfrac{\partial\varphi}{\partial\theta} & \dfrac{\partial\psi}{\partial\theta}\\ \dfrac{\partial\varphi}{\partial\omega} & \dfrac{\partial\psi}{\partial\omega}\end{vmatrix}=\left(\frac{\partial\psi}{\partial\omega}\right)^2\left(1+\left(\frac{d\omega}{d\theta}\right)^2\right),$$

whence

$$\frac{\partial\omega}{\partial\psi}\frac{\partial\psi}{\partial\omega}=\frac{1}{1+\left(\dfrac{d\omega}{d\theta}\right)^2}. \tag{2.5}$$

Substituting (2.4) into (2.1), and taking (2.5) into consideration, we obtain

$$\delta(\infty)=\delta(-\infty)+2V^2\int\limits_{\theta(\psi)}e^{-2\omega}d\theta, \tag{2.6}$$

where $\theta(\psi)$ is the trajectory of a particle in the plane $(\omega,\ \theta)$. The dependence $\Phi(\psi)_\infty$ will not appear as

$$\Phi(\psi)_\infty=\Phi(\psi)_{-\infty}+2V^2\int\limits_{\psi_*}^{\psi}\int\limits_{\theta(\psi)}e^{-2\omega}d\theta d\psi. \tag{2.7}$$

Let us consider a concrete example where there is steady-state symmetrical flow with a critical point during the collision of two jets of identical thickness, density, and velocity (Fig. 1). For steady-state flow with a critical point, the problem of finding the field of the velocities in the region of the flow has been solved [5]. The solution is written in the form

$$\begin{aligned}w&=\frac{V}{\pi}\left\{h_1\ln\left(1-\frac{\zeta}{a_1}\right)+h_2\ln\left(1-\frac{\zeta}{a_2}\right)-k_1\ln\left(1-\frac{\zeta}{b_1}\right)-k_2\ln\left(1-\frac{\zeta}{b_2}\right)\right\},\\ z&=\frac{V}{\pi}\left\{\frac{h_1}{a_1}\ln\left(1-\frac{\zeta}{a_1}\right)+\frac{h_2}{a_2}\ln\left(1-\frac{\zeta}{a_2}\right)-\frac{k_1}{b_1}\ln\left(1-\frac{\zeta}{b_1}\right)-\frac{k_2}{b_2}\ln\left(1-\frac{\zeta}{b_2}\right)\right\},\end{aligned} \tag{2.8}$$

where h_1, h_2, a_1, and a_2 are the respective thicknesses of oncoming jets at infinity; k_1, k_2, b_1, and b_2 are the same for expanding jets. In the actual example under consideration, we must set

$$\begin{gathered}h_1=h,\ a_1=Ve^{i\gamma},\ h_2=h,\ a_2=Ve^{-i\gamma},\\ k_1=h(1+\cos\gamma),\ b_1=V,\ k_2=h(1-\cos\gamma),\ b_2=Ve^{i\pi},\end{gathered} \tag{2.9}$$

where γ is the half-angle of the collision; V is the modulus of the jet velocity at infinity.

In one of the jets, for example, the jet C_1, let the originally straight liquid line $a-a$ (see Fig. 1), perpendicular to the free boundaries, be singled out. This means that, in (2.7), the term $\Phi(\psi)_{-\infty}\equiv 0$. With motion, the line $a-a$ will be deformed; part of it goes over into the jet C_3 and part into C_4. With removal to infinity, the parts of the line going over into expanding jets take on a definite form. Let us make a calculation of thê final form of the line going over, for example, into the jet C_3.

In the plane of the complex potential, the liquid line will move in the band $hV(1-\cos\gamma)/2 \le \psi \le hV(1+\cos\gamma)/2$ with a branch cut along the positive semiaxis $\psi=0$, along the lines ψ=const. In accordance with (1.4), (1.5), the line $a-a$ with $\varphi=-\infty$ is mapped by the vertical line φ=const and moves with the velocity V^2 in a positive direction, while its form in the plane w coincides with the form in the plane z, with an accuracy up to a normalizing constant. If the velocity ζ at any arbitrary point of the flow region is taken equal to $V\nu e^{-i\theta}$, then the relative velocity ζ at any arbitrary point (φ, ψ) is determined from the system of equations

$$\begin{gathered}\frac{2\pi\varphi}{hV}=\ln(1-2\nu\cos(\gamma-\theta)+\nu^2)+\ln(1-2\nu\cos(\gamma+\theta)+\nu^2)-\\-(1+\cos\gamma)\ln(1-2\nu\cos\theta+\nu^2)-(1-\cos\gamma)\ln(1+2\nu\cos\theta+\nu^2),\\ \frac{\pi\psi}{hV}=\operatorname{arctg}\frac{\nu\sin(\gamma+\theta)}{1-\nu\cos(\gamma+\theta)}-\operatorname{arctg}\frac{\nu\sin(\gamma-\theta)}{1-\nu\cos(\gamma-\theta)}-\\-(1+\cos\gamma)\operatorname{arctg}\frac{\nu\sin\theta}{1-\nu\cos\theta}+(1-\cos\gamma)\operatorname{arctg}\frac{\nu\sin\theta}{1+\nu\cos\theta}.\end{gathered}\tag{2.10}$$

The system (2.10) is obtained from the first equation of system (2.8) by separation of the real and imaginary parts, using the conditions (2.9). In the concrete case of a symmetrical collision, in (2.6) it is more convenient to go over to integration in the plane (ν, θ).

Since $e^{2\omega}=V^2\nu^2$, expression (2.6) assumes the form

$$\delta(\infty)=2\int\limits_{\Theta(\psi)}\frac{d\theta}{\nu^2}.\tag{2.11}$$

The trajectory $\Theta(\psi)$ can be represented in the plane (ν, θ). At the free boundary $\nu=1$ and $0\ge\theta\ge-\gamma$; therefore, the free boundary is represented by the segment AB in Fig. 2. Near the free streamline, $\nu<1$ everywhere, with the exception of the points $\varphi=\pm\infty$, and θ increases continuously from $-\gamma$ to 0; therefore, near the free boundary the streamline will be represented by the curve AEB. At the zero streamline in the interval $-\infty<\varphi\le 0$ there is a decrease in the angle θ from $-\gamma$ to $-\pi/2$, with a simultaneous decrease in ν from 1 to 0. This part of the line $\psi=0$ is represented by the curve AD in Fig. 2. Further, at the point 0 in the physical plane there is a rotation of the velocity vector by an amount $\pi/2$ $(-\pi/2\le\theta\le 0)$ with $\nu=0$, then a rise in ν from 0 to 1 with $\theta=0$. This part of the zero streamline is represented by the two segments DC and CB in the plane (ν, θ). The streamline near the line $\psi=0$ is represented by the curve AFB. Thus, any given trajectory $\Theta(\psi)$ in the plane (ν, θ) is represented by a curve with ends at the points A and B, lying in the curvilinear trapezoid ABCD.

For the form of the liquid line, taking account of (2.11) we obtain

$$\Phi(\psi)_\infty=2\int\limits_{\psi_0}^{\psi}\int\limits_{\Theta(\psi)}\frac{d\theta}{\nu^2}\,d\psi.\tag{2.12}$$

From (2.11) it follows that the liquid line is not perpendicular to the free boundary at infinity in the jet C_3:

$$\left.\frac{d\Phi}{d\psi}\right|_{\psi=\psi_0}=2\int\limits_{-\gamma}^{0}d\theta=2\gamma$$

or

$$\delta=\operatorname{arctg}2\gamma,\tag{2.13}$$

where δ is the angle between a perpendicular to the free boundary a'.

Specifically, for a collision angle $\gamma=\pi/2$

$$\delta=\operatorname{arctg}\pi\approx 72°,$$

and, for the breakthough of jets of a gradientless liquid

$$\delta=\operatorname{arctg}2\pi=81°.$$

To plot the form of the liquid line with $2\gamma=40, 50, 60°$ a numerical calculation was made of the integral in (2.12). The block of values of the integral of (2.11) was calculated, and then the dependence (2.12) was plotted using

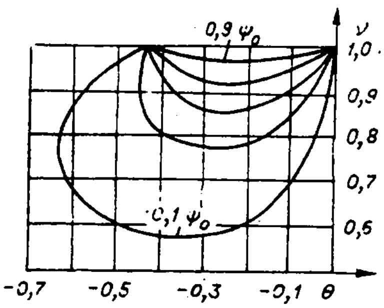

Fig. 3

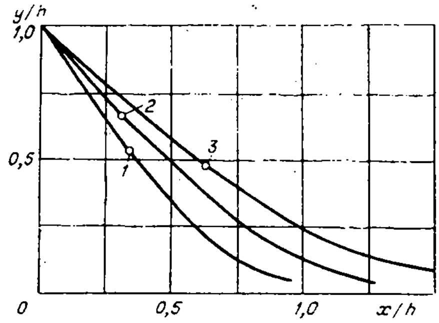

Fig. 4

the formula of a trapezoid. Calculation of the integral (2.11) started from the point $\nu = 1$, $\theta = -\gamma$. From an analysis of the second equation of system (2.10) it follows that $d\nu/d\theta$ at the point A (see Fig. 2) is determined as

$$d\nu/d\theta = \operatorname{ctg} \pi(\psi/hV + \cos\gamma/2). \tag{2.14}$$

Using (2.14), the values of $\nu_1 = \nu_0 + \Delta\nu$, $\theta_1 = \theta_0 + \Delta\theta$ were found, and the derivatives $\partial\psi/\partial\nu$, $\partial\psi/\partial\theta$ were calculated in accordance with the formulas

$$\frac{\partial\psi}{\partial\nu} = \frac{hV}{\pi}\left(\frac{\sin(\theta+\gamma)}{1-2\nu\cos(\theta+\gamma)+\nu^2} + \frac{\sin(\theta-\gamma)}{1-2\nu\cos(\theta-\gamma)+\nu^2} - (1+\cos\gamma)\frac{\sin\theta}{1-2\nu\cos\theta+\nu^2} + (1-\cos\gamma)\frac{\sin\theta}{1+2\nu\cos\theta+\nu^2}\right),$$

$$\frac{\partial\psi}{\partial\theta} = -\frac{hV}{\pi}\nu\left(\frac{\nu-\cos(\theta+\gamma)}{1-2\nu\cos(\theta+\gamma)+\nu^2} + \frac{\nu-\cos(\theta-\gamma)}{1-2\nu\cos(\theta-\gamma)+\nu^2} - (1+\cos\gamma)\frac{\nu-\cos\theta}{1-2\nu\cos\theta+\nu^2} - (1-\cos\gamma)\frac{\nu+\cos\theta}{1+2\nu\cos\theta+\nu^2}\right),$$

and the value of ψ using the second equation of system (2.10). If $|d\nu/d\theta|$, defined as

$$\frac{d\nu}{d\theta} = -\frac{\partial\psi/\partial\theta}{\partial\psi/\partial\nu},$$

was found to be greater than 1, then the calculation was made using the formula

$$\delta(\infty) = 2\int\limits_{\theta(\psi)} \frac{d\nu}{\nu^2 \frac{d\nu}{d\theta}}$$

with the constant spacing $\Delta\nu = d$, and the value

$$\Delta\theta = \Delta\nu \Big/ \frac{d\nu}{d\theta}.$$

In the contrary case, the calculation was made using the formula (2.11) with the constant spacing $\Delta\theta = d$ with $\Delta\nu = \Delta\theta d\nu/d\theta$. The spacing d was varied from 0.01 to 0.005. If the deviation $|\psi - \psi^0|$ (ψ^0 is a given streamline) was found not to be small, then for $|d\nu/d\theta| > 1$ a root was found for the equation $|\psi(\nu, \theta)| = \psi^0$ by the Newtonian method with ν = const, while for $|d\nu/d\theta| < 1$ the root was found with θ = const. The block of values of the integral (2.11) varied from 9 to 19.

Trajectories $\Theta(\psi)$ for $2\gamma = 50°$, calculated using the above-described method, are given in Fig. 3, which gives five curves of $\Theta(\psi)$ with values from $0.9\psi_0$ to $0.1\psi_0$, with the spacing $0.2\psi_0$. Figure 4 shows the form of the liquid line, calculated using (2.12) for the angles $2\gamma = 40$, 50, 60° (curves 1-3, respectively).

As can be seen from the results of the calculations, after passing through the region of the collision, the liquid line is not found to be perpendicular to the free surface, which is in agreement with (2.13). Close to the interface, the line $\Phi(\psi)$ [or, what is the same thing, x(y)] asymptotically approaches the axis $\psi = 0$ ($y = 0$). From a comparison of the results obtained with experiment, it is probably possible to judge approximately in what region of the real flow the state of the material is close to the state of a nonviscous incompressible liquid.

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VORTICAL MOMENTUM OF FLOWS OF AN INCOMPRESSIBLE LIQUID

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UDC 532.51

§1. In an incompressible liquid, filling the whole space outside of the finite region K (with a sufficiently smooth closed surface ∂K), let there be given a field of the velocities $\mathbf{v}(\mathbf{r}, t)$, satisfying the following conditions: a) sufficient smoothness; b) $\operatorname{div} \mathbf{v} = 0$; c) $|\mathbf{v}(\mathbf{r})| \sim \mathrm{const}/r^{1+\varepsilon}$, $|\mathrm{rot}\, \mathbf{v}| \sim \mathrm{const}/r^{4+\varepsilon}$ with $r \equiv |\mathbf{r}| \to \infty$ with small $\varepsilon > 0$.

We denote by $\mathbf{n}$ the external normal to ∂K, and by G the region filled with the liquid. Let the liquid density $\rho = 1$. It is convenient to use the following representation for $\mathbf{v}(\mathbf{r}, t)$;

$$\mathbf{v}(\mathbf{r}) = \mathrm{grad}\, \varphi + \mathrm{rot}\, \mathbf{A}; \tag{1.1}$$

$$\begin{cases} \varphi(\mathbf{r}) = -\dfrac{1}{4\pi} \oint\limits_{\partial K} \dfrac{\hat{\mathbf{n}} \cdot \hat{\mathbf{v}}}{s}\, dS, \\ \mathbf{A}(\mathbf{r}) = \dfrac{1}{4\pi} \left[\int\limits_G \dfrac{\hat{\boldsymbol{\omega}}}{s}\, dV + \oint\limits_{\partial K} \dfrac{\hat{\mathbf{n}} \times \hat{\mathbf{v}}}{s}\, dS \right], \end{cases} \tag{1.2}$$

where $s = |\mathbf{r} - \hat{\mathbf{r}}|$; the symbol "$\wedge$" denotes variables with respect to which integration is carried out; $\omega \equiv \mathrm{rot}\, \mathbf{v}$.

This representation for the case of a finite G is given in [1]; for an infinite region G it is proved by a direct calculation [substitution of (1.2) into (1.1)], taking account of limitations on the asymptotic of the field of the velocity. Here a formal calculation gives $\mathbf{v}(\mathbf{r}) \equiv 0$ in the region K outside the liquid.

The latter shows that the flow in G can be integrated as the flow in the whole space, obtained by "filling" of the region K with a liquid at rest. Under these circumstances, at ∂K there is a discontinuity of both the tangential and normal components of the velocity, corresponding to the distribution (1.2) of the vortices of the density $\mathbf{n} \times \mathbf{v}$ and sources of the density $\mathbf{n} \cdot \mathbf{v}$ at ∂K.

Naturally, the region K can be filled in any other arbitrary way (not necessarily by a liquid at rest); under these circumstances, there is a change in the distribution of the vortices and sources in (1.2).

However, the representation (1.2) has the advantage that the "filling" of K with a liquid at rest does not change the total momentum of the flow, which will be important in what follows with a generalization of the concept of momentum.

§2. Momentum of Flows of an Incompressible Liquid. The usual definition of the momentum of a flow, which we shall call the "true" momentum I, has the form

$$\mathbf{I} = \int\limits_{V_0} \mathbf{v}\, dV, \tag{2.1}$$

where V_0 is the volume occupied by the liquid; $\mathbf{v} = \mathbf{v}(\mathbf{r}, t)$ is the field of the velocity.

This definition is applicable for both finite and infinite V_0, with the condition of the absolute convergence of the integral (2.1).

However, in the case of a liquid filling the whole space outside some limited system of bodies, the integral (2.1), for flows having a dipolar asymptotic, does not converge absolutely. Its value is found to depend on the manner in which the volume of the integration approaches infinity. In the case of absolute convergence of (2.1) for a liquid filling the whole space, $\mathbf{I} = 0$; i.e., for all such flows with a zero momentum, the definition (2.1) has no meaning.

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